STAT 651 Midterm Test (60 minutes) - 2:20pm - 3:20pm October, 2015

NAME: Solutions Total number of Marks: /25

There are 7 questions in this paper, do not be deterred, they are all straightforward. Read each question carefully. There are questions on both side of the page. The number of marks for each question is given in brackets. Be smart about how you answer. If you can't answer one question move on to the next and return to the questions you could not do after answering all the other questions! There are 6 Figures in this paper.

Rubric

(1) Do each test at the 5% level.

(2) No marks will be given for answering only reject null or don't reject null. Explain your conclusion by stating your answer with respect to the p-value or rejection region.

(3) If you need to do a calculation then give it.

(4) This exam is a closed book exam, but you can use a 3-sided cheat sheet, normal, t-tables and a calculator.

Write your solutions in the question paper.

GOOD LUCK!

(1) (a) For a given data set, the sample variance is zero, what can we conclude about the numbers in the data set? [1]

All data is ~~the~~ takes same value.

(b) For a given data set, the Interquartile Range (IQR) is zero, what can we conclude about the data set? [1]

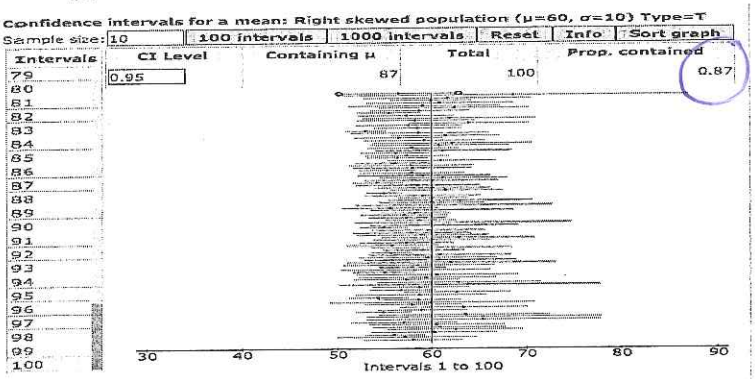
At least 50% data (about the median) is the same

(c) You are given the hypotheses $H_0 : \mu \geq 5$ vs $H_A : \mu < 5$ and population variance $\sigma^2 = 1$. A sample of size 9 is drawn and the sample mean is $\bar{x} = 6$, what is the conclusion of the test at the 5% level? [1]

Since $\bar{x} = 6 \geq 5$, we cannot reject the null.

p-value > 50%.

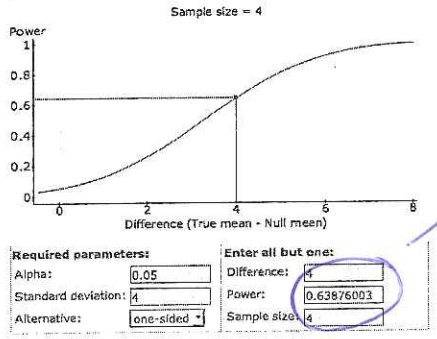
(2) (a) A student uses the t-distribution to construct a 95% confidence interval. To determine if one he really has 95% confidence in the interval he runs the following applet. Use the applet to determine how much confidence he really has in the interval. [1]



We only have about 87% confidence in the interval.

(b) Gestational diabetes is diagnosed in an expectant mother if her mean level is over 140. We test the hypothesis $H_0 : \mu \leq 140$ vs $H_A : \mu > 140$ and do the test below. To test this hypothesis we note that the standard deviation of a blood sample is $\sigma = 4$, and the average of four blood samples are taken

A person is said to have moderate gestational diabetes if their mean level is $\mu = 144$. Use the applet below to assess the test's ability to detect a person who has moderate gestational diabetes. [1]



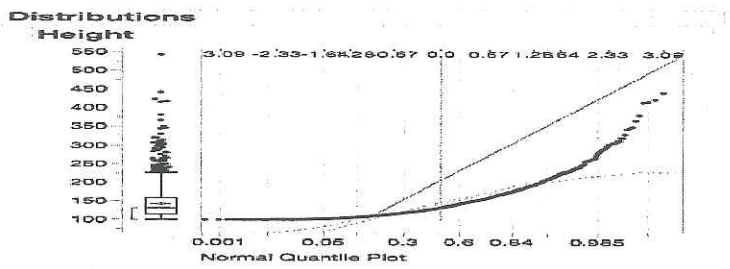
Power = 63.8%

Hence the chance of rejecting the null when she has moderate diabes = 63.8%.

Therefore

The test is not very sensitive to moderate diabetes.

(3) A QQplot of the height of skyscrappers is below.



(a) Describe the main feature of the distribution. Right Skewed. [1]

(b) Make a plot of the distribution of the sample mean for sample sizes $n = 5$, $n = 18$ and $n = 200$. Plot them next to each other. Assuming the mean height of a skyscraper is $\mu = 120m$ and the standard deviation is $\sigma = 40m$ for each plot give the mean, standard error and an indication of the shape of the distribution. [2]

$n=5$ → Right skewed

$s.e = \frac{40}{\sqrt{5}}$ $\mu = 120$

$n = 18$ → still quite Right skewed

$s.e = \frac{40}{\sqrt{18}}$ $\mu = 120$

$n = 200$ → close to normal.

$se = \frac{40}{\sqrt{200}}$ $\mu = 120$

(e) Given the output below construct a 95% confidence interval for the mean (remember to use a t-distribution). Comment on whether we really have 95% confidence that the interval contains the mean? [2]

Distributions

Height

Summary Statistics

Mean	362.95556
Std Dev	63.510483
Std Err Mean	14.969564
Upper 95% Mean	394.53858
Lower 95% Mean	331.37254
N	18

$[362 \pm 2.11 \times 14.9]$

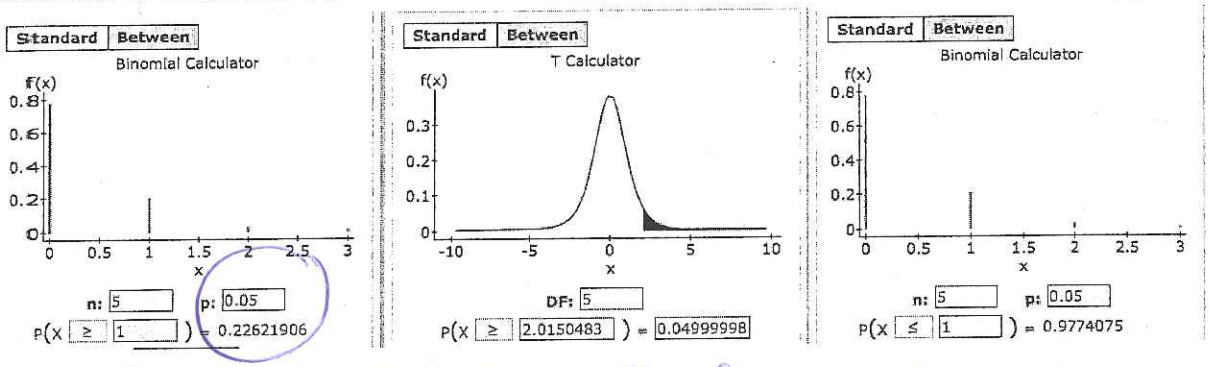
Since the data is right skewed, the average based on 18 won't be very close to normal. Therefore we will have less than 95% confidence in the interval.

(4) A researcher does 5 statistical tests, each test is done at the 5% level.

(a) Suppose the null is true, using the plots below what is the chance of falsely rejecting the null **at least** one out of 5 times? [1]

$Bin(0.05, 5)$

$P(S_5 \geq 1) = 0.226$



There is a 22.6% of rejecting the null falsely at least one out of five times when the null is true.

(b) Using your answer from (a), explain why testing a hypothesis multiple times can lead to misleading conclusions? (2 lines max.) [1]

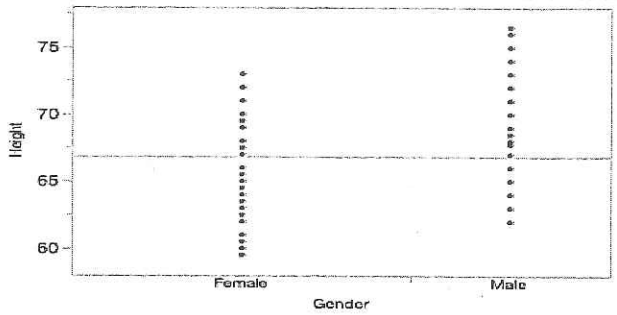
We see from (a) that when test multiple times we can 'spuriously' reject the null. Beware! with a large chance we do.

(5) You read the following excerpt from a newspaper 'In a sample of 300 randomly sampled students, 52% had a part time job. It is clear from this this sample that the majority of students have part time jobs.'

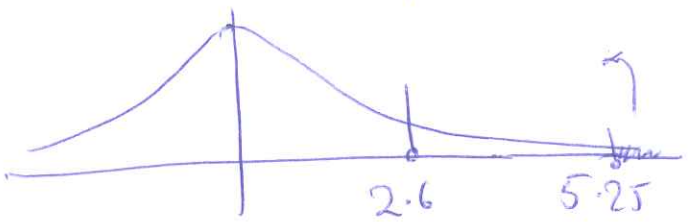
Explain in the context of sample and population, why we cannot **immediately draw** this conclusion based on the sample? (3 lines max.) [2]

This is only a sample from the population of students. It can be that a sample gives a proportion of 52% whereas the population as whole only as proportion who work that is 50% or less. We need to calculate the likelihood of this happening — this is a p-value.

Oneway Analysis of Height By Gender



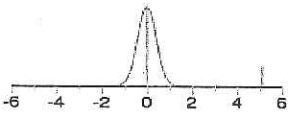
p-value $< 0.01\%$.



t Test

Male-Female
Assuming unequal variances

Difference	5.12865	t Ratio	12.66273
Std Err Dif	0.40502	DF	160.3381
Upper CL Dif	5.92851	Prob > |t|	<.0001*
Lower CL Dif	4.32879	Prob > t	<.0001*
Confidence	0.95	Prob < t	1.0000



probability	0.15	0.10	0.05
t^*	1.04	1.28	1.65
probability	0.025	0.01	0.005
t^*	1.97	2.35	2.60

(6) 160 males and females (combined) were randomly sampled and their heights recorded in inches. The data is summarized in the JMP output above. The t-tables for a t-distribution with 160.33 df is also given above.

(a) What is the average difference in male and female heights of this sample? [1]

5.128

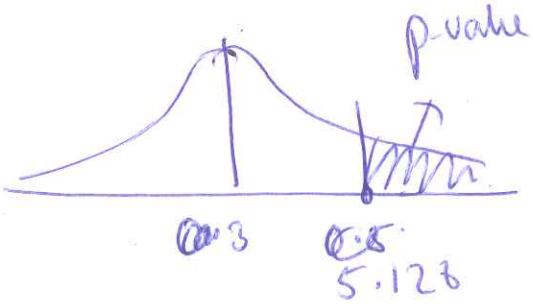
(b) Suppose you want to test the hypothesis that males are taller, what is the hypothesis of interest? Use the output above to do the test at the 5% level. [2]

$H_0: \mu_M - \mu_F \leq 0$ $H_A: \mu_M - \mu_F > 0$.

p-value $< 0.01\%$ reject null.

(c) Use the output above to test the hypothesis that **males are on average more than 3 inches taller than females.** State the hypothesis of interest and the conclusion of the test. [2]

$H_0: \mu_M - \mu_F \leq 3$ $H_A: \mu_M - \mu_F > 3$



$t = \frac{5.128 - 3}{0.405} = 5.25$

p-value $< 0.01\%$ reject null.

(d) One researcher removed the sample mean from each of the two groups (these are called residuals) and used the **residuals** to test the hypothesis in (b). Explain why the result of this test is meaningless (2 lines max.). [1]

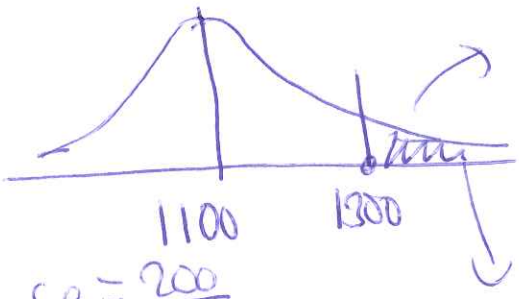
residuals centralize the data in each group. Sample mean of residuals in each group is zero. Of course we cannot reject null! Residuals are only used for checking model assumptions.

(7) The grades in an exam are normally distributed with a (population) mean $\mu = 1100$ and (population/known) standard deviation $\sigma = 200$.

(a) In a class of 10 students, what is the distribution of the sample mean (give the mean the standard error and the distribution). [1]

$\bar{X} \sim N\left(1100, \text{s.e} = \frac{200}{\sqrt{10}}\right)$ $\hookrightarrow \frac{200^2}{10}$.

(b) What is the chance that the class average (in a class of 10) will be **1300 or over** (make a plot)? [2]



$z = \frac{1300 - 1100}{200/\sqrt{10}} = 3.16$

look up tables

$se = \frac{200}{\sqrt{10}}$

$0.0008 \approx 0.08\%$

(c) Astar coaching **randomly samples** 10 students from the population of students who will take the exam. After coaching the average in this class is 1300, using the information in (b), what can we say about Astar coaching? State the hypothesis of interest and do the test at the 5% level. [2]

$H_0: \mu \leq 1100$ $H_A: \mu > 1100$

The p-value = $0.08\% < 5\%$ evidence to reject null and suggest Astar is doing helping students.