STAT 651 Final Exam (2 hours, 15 minutes) December, 2014

Solutions

NAME: Total number of Marks: /40

There are 8 questions in this paper, do not be deterred, they are straightforward. Read each question carefully. There are questions on both side of the page. The number of marks for each question are given in brackets. Be smart about how you answer. If you can't answer one question move on the to next and return to the questions you could not do after answering all the other questions! There are 4 Figures in this paper. You must submit a cheat sheet with this exam.

DO ALL TESTS IN THIS PAPER AT THE 5% LEVEL.

Rubric: This exam is a closed book exam, but you can use 4-sides of cheat sheet, normal tables, t-tables, both Wilcoxon tables, chi-square tables, scrap paper and a calculator. You can also use an English dictionary, but these must be declared to me (so I can check that there is nothing inside).

(*) If not stated otherwise give 95% CI

(*) Q6 → give probabilities.

Write your solutions in the question paper.

GOOD LUCK!

9 + 8 + 5 + 5 + 5 + 8

Spare page.

(1) (i) The standard deviation for the size of tomato plants is known to be between $5-10$cm. A horticulturist wants to estimate the mean height of a tomato plant. How large a sample size should she collect to be sure that the margin of error is maximum 0.5cm? [2]

@a 95CI.

$$\text{MOE} = 1.96 \times \frac{6}{\sqrt{n}} = 1.96 \times \frac{10}{\sqrt{n}} = 0.5$$

$$\Rightarrow n = \left(\frac{1.96 \times 10}{0.5}\right)^2$$

(ii) You are given the data set

1, 1.2, 2, 3, 3.5, 3.2, 4 4.5 4.6

Suppose 4.6 is increased subtantially. By answering, increase, decrease or stay the same, what will happen to: [3]

* Mean — Increase
* Median — Stay Same
* First quartile. — Stay Same

(2) A study was conducted to see whether Human intelligence is any different to that of Pig or a Dog. To see whether there was a difference 100 human volunteers, 90 Pig 'volunteers' and 110 dog 'volunteers' were given a puzzle to solve (with a treat if they correctly solve it). The results are summarized below.

	Solved	Number of participants
Humans	70	100
Pigs	55	90
Dogs	50	110

(i) Suppose there is no difference between human, pig and dog intelligence. What is the best estimate for the probability a puzzle is correctly solved? [2]

$$\frac{70+55+50}{100+90+110} = \frac{175}{300} = 0.875 \; 0.583$$

(ii) Suppose there is no difference between the intelligence of a pig and dog, what is the best estimator for the probability that either a pig or dog can solve the puzzle? [2]

$$\frac{55+50}{90+110} = \frac{105}{200} = 0.525$$

9

(3) It is known that the probability that the birth weight of a human baby is over 12 pounds is 0.01 (P(new born baby weight > 12) $= 0.01$), however the probability that a baby is over 12 pounds given that his or her sibling is over 12 pounds is 0.5 (P(new born baby weight $> 12|$ sibling weight > 12) $= 0.5$).

(i) Recently, while visiting a friend at a maternity ward, I found two unrelated babies, both born on the same day, were over 12 pounds. Calculate the probability of this happening. [2]

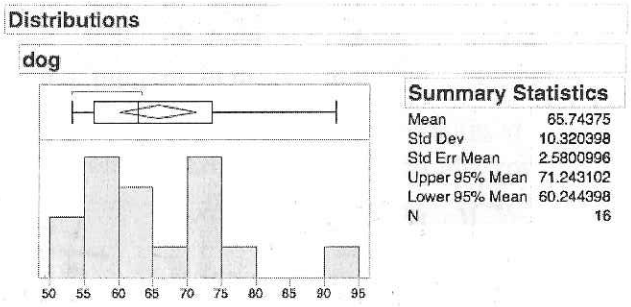
$(0.01)^2$

(ii) A local newspaper reported that the birth weight of both of Maria's children, now age 1 and 3, was over 12 pounds. And remarked that this was a remarkable event! Calculate the probability of this happening, is this event as remarkable as the newspaper claims? [2]

$(0.01) \times (0.5) = 0.005. = 1/2\%$

which is a small probability but not 'remarkably' small.

(4) Recently, there have been concern that the proportion of obese dogs is rising. It is known that the mean weight of mid-size dogs used to be 70 pounds (before the year 2000). A random sample of 16 mid-size dogs is collected. The data is summarized below. Use the t-distribution.



(i) Suppose we want to investigate the conjecture that the mean is now over 70 pounds. State precisely the hypothesis of interest and do the test at the 5% level. Is there any evidence to back the claim? [2]

$H_0: \mu \leq 70 \quad H_A: \mu > 70$

Cannot reject the null.

(ii) Construct a 99% confidence interval for the mean weight of a mid-size dog. [2]

2.947

$[65.74 \pm t_{15}(0.005) \times 2.58]$

8

(5) It appears that many other animals, besides us, have traditions.

An article was recently published in Nature (3rd December 2014) on the diffusion of animal behaviours through animal populations and how these behaviours were kept over time. The focus was on the learning patterns of great tits (a type of bird). Two different woods (where these birds live, note that a wood is similar to a forest) were under investigation. We name them Wood A and Wood B. Four great tits, two from Wood A and two from Wood B, were captured. Over a period of two days the two great tits from Wood A were 'taught' how to remove a mealworm (considered tasty by great tits) from a box. Over the same time period the two great tits from Wood B were shown the box, however no attempt was made to show how to remove the mealworm inside.

After two days training period, the birds were reintroduced back to their respective woods. The boxes with mealworms inside was placed over the woods and observed over a period of 20 days. The number of birds which attempted to remove the worm was counted as well as the number of successful attempts. The data is here:

	No. who succeed	Total no.
Wood A	80	96
Wood B	24	46

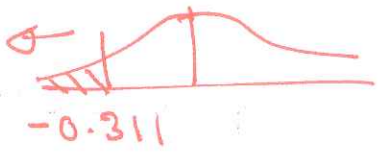
Hypothesis test results:
p_1 : proportion of successes for population 1 B
p_2 : proportion of successes for population 2 A
$p_1 - p_2$: Difference in proportions
$H_0 : p_1 - p_2 = 0$
$H_A : p_1 - p_2 < 0$

Difference	Count1	Total1	Count2	Total2	Sample Diff.	Std. Err.
$p_1 - p_2$	24	46	80	96	-0.3115942	0.079387068

Figure 1: Bird Data

(i) The researchers want to investigate whether teaching a few birds helped increase the chance of information diffusing (going through) through the population. State the hypothesis of interest and do the test at the 5% level. [3]

$H_0 : p_B - p_A \geq 0$ $H_A : p_B - p_A < 0$



$Z = \frac{-0.311}{0.079} = -3.936$

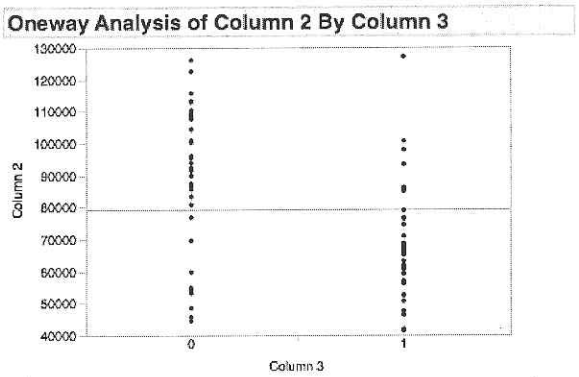
pvalue less than 0.0002.

(ii) Given the standard error 0.083 calculate a 95% confidence interval for the difference in success rates. [2]

$[-0.311 \pm 1.96 \times 0.083]$

5

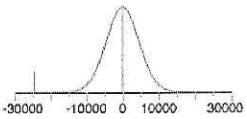
(6) A company is under investigation for discrimination. The main allegation is that it pays its female employees less than male employees. The salaries of its female and male employees are collected. The data and test results are summarized in Figure 3. Note that 0 = Male and 1= Female.



t Test

1-0
Assuming unequal variances

Difference	-24411	t Ratio	-5.45919
Std Err Dif	4471	DF	73.38893
Upper CL Dif	-15500	Prob > |t|	<.0001*
Lower CL Dif	-33321	Prob > t	1.0000
Confidence	0.95	Prob < t	<.0001*



Wilcoxon / Kruskal-Wallis Tests (Rank Sums)

Level	Count	Score Sum	Expected Score	Score Mean	(Mean-Mean0)/Std0
0	40	2090.00	1620.00	52.2500	4.518
1	40	1150.00	1620.00	28.7500	-4.518

Figure 2: Gender

(i) State the hypothesis of interest. [1]

$H_0: \mu_1 - \mu_0 \geq 0, H_A: \mu_1 - \mu_0 < 0$

(ii) Do an independent sample t-test at the 5% level. [2]

p-value < 0.0001 Hence reject the null. There is evidence women are key paidless.

(iii) A few outliers were observed, therefore test the same hypothesis using the Wilcoxon sum-rank test.

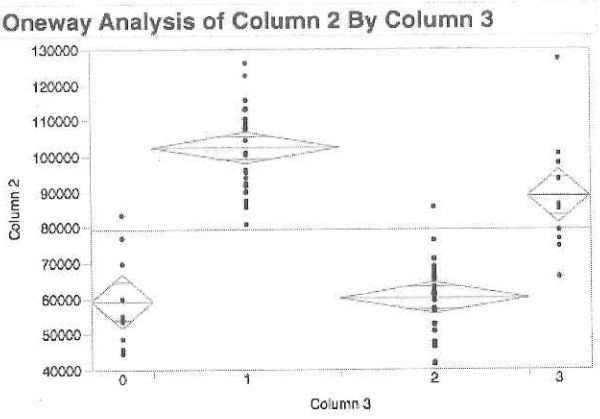
Note: Table 5 does go up to sample size $n_1 = 40$ and $n_2 = 40$. Therefore we give the relevant values for $n_1 = 40$ and $n_2 = 40$. [2]

$\alpha = 0.025$ one sided, $\alpha = 0.05$ two sided	$T_L = 1416$	$T_U = 1824$
$\alpha = 0.05$ one sided, $\alpha = 0.1$ two sided	$T_L = 1450$	$T_U = 1790$

If the alternative is true then $S = 1150$ will be below the 5% threshold. Since $1150 < 1450$ we reject the null at a 5% level

5

(7) During the investigation of the above company, more information has come to light. In particular, information about the ages of the employees. The data is now grouped in terms of gender and whether the employee is above or below 40 years old. 0 =male and below 40, 1 = male and above 40, 2 = female and below 40, 3 = female and above 40.



Missing Rows 3

Oneway Anova

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Column 3	3	3.21e+10	1.07e+10	73.8733	<0.0001*
Error	76	1.1008e+10	144842909		
C. Total	79	4.3108e+10			

Figure 3: Gender and Age

(i) Suppose we are interested in seeing whether salary depends on either age/and or gender. State the hypothesis of interest and the result of the test. [3]

H_0: All the means are the same H_A: At least one means diff

p-value < 0.05%. Hence reject the null at the 5% level.

(ii) Using the above data, state the hypothesis and the test that should be done to see whether females are being paid less than their male counterparts. [2]

$H_0: \mu_{JF} - \mu_{JM} \geq 0$ $H_A: \mu_{JF} - \mu_{JM} < 0$

$H_0: \mu_{SF} - \mu_{SM} \geq 0$ $H_A: \mu_{SF} - \mu_{SM} < 0$

Either an independent sample t-test or a Wilcoxon sum rank Test

5

(8) Every Wednesday night a group of A&M students enjoy playing Dungeons and Dragons. An important component in the game is a 6 sided die (a cube with one number on each of the six sides). There have been claims that Dungeon master is using a weighted die (a die which is not fair and does not give equal weight to each number). To see whether there is any evidence of this, the die is thrown 120 times and the outcomes recorded.

Side of die	1	2	3	4	5	6
Number of outcomes	12	20	21	18	24	25
	20	20	20	20	20	20

(i) Suppose the die is fair how many of each number would you expect to see. [1]

(ii) State the hypothesis of interest. [1]

H_0: chance of rolling any number $= 1/6$

H_A: chance of rolling ~~any~~ a number (at least one) $\neq 1/6$

(iii) The T-value in the test is 5.5, what are the results of the test at the 5% level (giving bounds for the p-value). [2]

$T = 5.5 \quad X^2_5(0.05) = 11.07$ Since $5.5 < 11.07$

Cannot reject null

pvalue is 10-90%.

(iv) Suppose the outcome in part (iii) is that we cannot reject the null, does this prove that the die is fair (give a reason for your answer)? [2]

No, it simply means the data is consistent with the null being true. If we had a much larger sample size we could detect differences.

(v) Suppose the T-value in the test is 0, what is the corresponding p-value and what is special about what is observed and what we expect? [2]

The p-value = 100% and the observed and expected values are the same.

8