Solutions

STAT 651 Final Test (2 hours) December 9th, 2008

NAME: Total number of Marks: /~~35~~ 40

Answer all the questions in the exam (questions are on both sides of the paper). There are 0+ 9 questions in this paper.

Advice: Look at the marks allocated for each question and don't spend a disproportionately large amount of time on any one question.

When conducting a test, always state your null and alternative. Also state the distribution and/or the test that you use.

Write your solutions in the question paper.

(0) What is your first name? [3]

(1) I draw 200 samples each of size 50. For each sample I construct a 99% CI. On average how many of the CIs will be in <u>error</u>? [2]

2

(2) I draw an independent sample of size 51 from a population. The sample mean is 8 ($\bar{X} = 8$) and the sample variance is 4. Suppose my research hypothesis is that the population mean (μ) is greater than 9. State the null and alternative hypotheses. What are the conclusions of test at the 5% level? [2]

/

$H_0: \mu \leq 9 \quad H_A: \mu > 9$

$\bar{X} = 8$ not enough evidence to reject null.

(3) Xuan and Xi are having a discussion about how to choose the sample sizes in order to compare the means in an independence sample t-test.

Xuan says that choosing two samples each of size 80 would lead to a better estimator of the mean difference.

On the other hand Xi thinks that choosing two samples one of size 30 and the other of 200 would be better.

Who is correct, give a reason for your answer? [2] 3

Compare $\frac{1}{30} + \frac{1}{200} = 0.0433$ with $\frac{1}{80} + \frac{1}{80}$ 0.025.

and look for the smaller value

Xuan.

(4) Look at the following partially completed ANOVA table.

	Sum of Squares	df	Mean square	F	Sig.
Between Groups	80	5	80/5 = 16	7.9	
Within Groups	200	99	200/99 = 2.02		
Total	280	1			

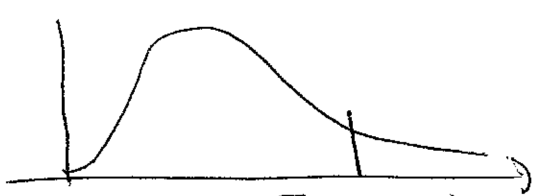
(a) How many groups (populations) have been sampled? [1]

6

(b) Test the hypothesis H_0 : all the means are the same against H_A : At least one of the means are different. [2] 3

Useful information: $F_{5,99}(0.05) = 2.306$ $F_{99,5}(0.05) = 4.40$.

$$\frac{80/5}{200/99} = 7.9$$



Since $7.9 > 2.306$

enough evidence to reject null.

(5) SPSS is used to do a paired t-test. Part of the output is given below

	Mean Difference	Std. Deviation	Std. Error Mean	95% Confidence interval Lower	Upper
Pair 1	2	1.1	0.1717911	1.656	2.344

(a) Using the information above, deduce the sample size. [2]

$$\frac{1.1^2}{n} = 0.171^2 \Rightarrow n = \frac{1.1^2}{0.171} = 41$$

(b) Using the sample size evaluated in (a). If you did not find one use $N = 31$.

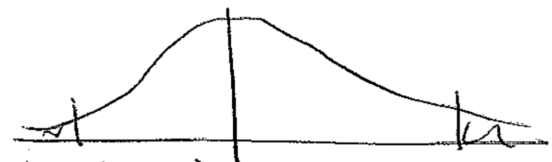
(i) Suppose you wanted to test the hypothesis that the means in the populations were different. State the null and alternative and the conclusions of the test (do the test at the 5% level). [3]

$H_0: \mu_1 - \mu_2 = 0$

$H_A: \mu_1 - \mu_2 \neq 0$

make a t-transform:

$$\frac{2-0}{0.1719} = 11.6$$



$-t_{40}(0.025)$ $t_{40}(0.025)$

$-t_{30}(0.025)$ $t_{30}(0.025)$

Compare with

$t_{40}(0.025) = 2.021$

$t_{30}(0.025) = 2.042$

Since $11.6 > 2.021\ (2.042)$

enough evidence to reject null.

(ii) Constuct a 90% CI for the mean difference. [2]

$$2 \pm t_{40}(0.045) \times 0.1717911$$

$t_{40}(0.05) = 1.684$

$t_{30}(0.05) = 1.690$

$$2 \pm 1.684 \times 0.1719$$

$$[1.71, 2.29]$$

(6) Elections in the United Burrows of Aardvarks (UBA) are due to take place next week. There are two candidates, their names are Ernie and Bert.

There has been some speculation that Ernie may win the elections. To test this hypothesis a poll of 1000 aardvarks was taken and the data is summarised below.

	Ernie	Bert
number	600	400

(a) What test would you use to test the hypothesis that Ernie will win? [1]

Test on the proportions.

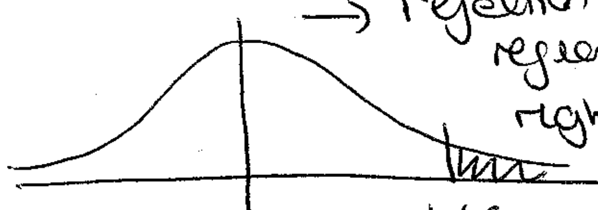
(b) Based on this sample, is there evidence to suggest that Ernie will win? Remember, to state clearly the null and alternative (do the test at the 5% level). [8] 9

~~Hyp~~ Let π = proportion of Aardvarks who would vote for Ernie

$H_0: \pi \leq 0.5 \quad H_A: \pi > 0.5$

$\hat{\pi} = 0.6$. Z-transform:

$$z = \frac{0.6 - 0.5}{\sqrt{\frac{0.6 \times 0.4}{1000}}} = \frac{0.1}{\sqrt{\frac{0.24}{1000}}}$$



→ rejection region on right.

$$= \frac{0.1}{0.015} = 6.6 \quad \text{since} \quad 6.6 > 1.64$$

enough evidence to reject null.

(7) We return to the United Burrows of Aardvarks elections. There is a belief that the more northern Aadvarks are more likely to vote for Ernie than southern Aardvarks. A survey was done and Aardvarks were asked their geographical location (North, Mid or South) and voting preference (Ernie or Bert). The data is summarised below.

	North	Mid	South	
Ernie	250	150	200	600
Bert	50	150	200	400
	300	300	400	1000

(a) State the test you would use to do see whether there is a relationship between location and voting habits. [1]

χ^2 test for independence

(b) Stating precisely the null and alternative, do the test at the 5% level.
Is there evidence to suggest that Northern Aardvarks voting habits are different to more southern Aardvarks? [3]

[4]

H_0: location and candidate are independent.

H_A: location and candidate are not independent.

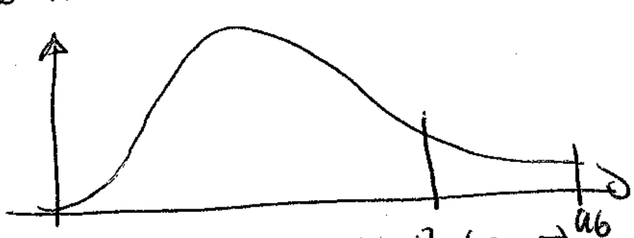
Expected values of independent

	North	Mid	South
Ernie	$0.6 \times 300 = 180$	180	240
Bert	$0.4 \times 300 = 120$	120	160

$$T = \frac{(250-180)^2}{180} + \frac{(180-150)^2}{180} + \frac{(200-240)^2}{240} + \frac{(150-120)^2}{120} + \frac{(150-120)^2}{120}$$

$$+ \frac{(160-200)^2}{160} = 27 + 5 + 6.6 + 40.8 + 7.5 + 10 = 97.2$$

Under the null T has a χ^2 with 2 degree of freedom



$\chi^2_2(0.05) = 5.991$

Since $96 >> 5.991$

Enough evidence to reject null.

(8) There has been a lot of speculation that the use of cell phones while driving has increased the number of accidents.

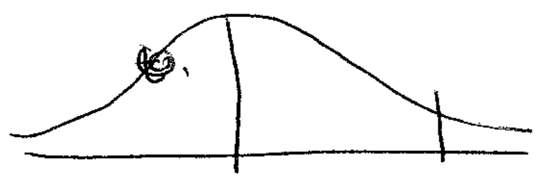
Scientists wanted to test whether talking on a cell phone increased a drivers reaction time. To test the hypothesis, they randomly sampled 30 people and placed each of them in a car simulator. For each driver the reaction time to the sudden appearance of a colour stimuli when the driver was <u>not</u> on the the phone and when the driver was <u>on</u> the phone was recorded.

Based on this sample the average reaction time when <u>not</u> on the phone was $\bar{X} = 0.5$ seconds. The average response time when <u>on</u> the phone was $\bar{Y} = 0.7$ seconds. The pooled sample variance is $s_p^2 = 0.4^2$ and the sample variance of the differences is $s_d^2 = 0.1^2$.

(i) Is there evidence to suggest that using a cell phone increases the reaction time while driving (state the test you would use, the hypothesis and do the test at the 5% level)? [4]

$H_0: \mu_p - \mu_0 \leq 0$ $H_A: \mu_p - \mu_0 > 0$. Do a paired t-test.

$$\frac{0.7-0.5}{\sqrt{0.4^2/30}} = \frac{0.2}{0.073} = 11.1$$



$12.7 > 1.699$ enough evidence to reject null. $t_{29}(0.05) = 1.699$

More space

(ii) Suppose there are several outliers in the data and a QQplot suggests a large deviation from normality, what test would you use instead? [1]

Wilcoxon-sign rank test

(9) A dating agency wants to investigate whether there is a correlation between the heights of couples. They surveyed 52 (male/female) couples. Let X denote the height of females and Y the height of males.

From this data it is found that the average female height is $\bar{X} = 1.7$ meters and the average male height is $\bar{Y} = 1.9$ meters. $S_{XX} = \sum_{i=1}^{52}(X_i - \bar{X})^2 = 10$, $S_{XY} = \sum_{i=1}^{52}(X_i - \bar{X})(Y_i - \bar{Y}) = 8$, $S_{YY} = \sum_{i=1}^{52}(Y_i - \bar{Y})^2 = 12$.

Is there evidence to suggest that ~~there is~~ a positive correlation between the heights of partners? (State ~~the~~ null and alternat do the test at 5% level) [4]

is greater than 0.5

The estimate of the correlation coefficient is

$H_0: \rho_{XY} \leq 0.5$

$H_A: \rho_{XY} > 0.5.$

$$\hat{r}_{XY} = \frac{S_{XY}}{\sqrt{S_{XX} S_{YY}}} = \frac{8}{\sqrt{10 \times 12}} = \frac{8}{10.954} = 0.730$$

The s.e is $\sqrt{\frac{1-\hat{r}_{XY}^2}{n-2}} = \sqrt{\frac{1-0.533}{50}} = 0.096$

$H_0: \rho_{XY} \leq 0.5 \quad H_A: \rho_{XY} > 0.5$

$t_{50}(0.05) = 1.676.$

$$t = \frac{0.730 - 0.5}{0.096} = 2.3$$

2.3 > 1.676 enough evidence to reject null.