Solutions

Midterm 3 - STAT 303
Fall 2021

Name:

UIN:

Signature:

1. Do not open this test until told to do so.
2. Please **read** each question carefully.
3. This is a closed book examination, However you may use the cheat sheet provided and the tables you have brought with you. You should have no other printed or written material with you on the exam. But scrap paper is allowed.
4. You have 60 minutes to work on this exam. There are **15** multiple choice questions.
5. You may use a calculator in the exam.
6. If there is no correct answer or if multiple answers are correct, select the **best** answer.
7. If you are unsure of what a question is asking for, do not hesitate to ask the instructor or course assistant for clarification.
8. Do all tests at the **5%** level unless specified otherwise.
9. Some questions are very easy (don't make them more complicated than they are), make sure you get these correct.
10. Good Luck!!!

(1-4) Researchers are trying to understand if doing boring tasks **increases** creativity. To test this conjecture 60 volunteers were randomly placed into two different groups (each of size 30); Group 1 or 2.

- Group 1 was given the task of reading the telephone numbers in the phone book for 20 minutes (this is considered a very boring activity).
- Group 2 was given the task of reading a novel for 20 minutes (this is not considered a boring activity).

After the task, both groups were asked to write down the number of different things they could do with a paper cup. For each individual in each group the **number** of different things was recorded. The results are summarized below.

Two sample T hypothesis test:
μ_1 : Mean of Telephone
μ_2 : Mean of Novel
$\mu_1 - \mu_2$: Difference between two means
$H_0 : \mu_1 - \mu_2 = 0$
$H_A : \mu_1 - \mu_2 > 0$
(without pooled variances)

Hypothesis test results:

Difference	Sample Diff.	Std. Err.	DF	T-Stat	P-value
$\mu_1 - \mu_2$	0.96666667	0.36288541	57.485992	2.6638344	0.005

(1) The researchers are investigating if **boring** activities increase the level of creativity. Let $\mu_{telephone}$ denote the mean number things a person thinks of doing with a paper cup after reading the phone book and μ_{novel} denote the mean number of things a person thinks of doing with a paper cup after reading a novel. What is the hypothesis of interest and the result of the test based on the data? Do the test at the 5% level.

(A) $H_0 : \mu_{telephone} - \mu_{novel} \leq 0$ vs $H_A : \mu_{telephone} - \mu_{novel} > 0$, the p-value is 0.5%. The data suggests that doing boring activities **decreases** creativity.

(B) $H_0 : \mu_{telephone} - \mu_{novel} \leq 0$ vs $H_A : \mu_{telephone} - \mu_{novel} > 0$, the p-value is 0.5%. The data suggests doing boring activities **increases** creativity.

(C) $H_0 : \mu_{telephone} - \mu_{novel} \geq 0$ vs $H_A : \mu_{telephone} - \mu_{novel} < 0$, the p-value is 99.5%. There is evidence to suggest that doing boring activities **decreases** creativity.

(D) $H_0 : \mu_{telephone} - \mu_{novel} \leq 0$ vs $H_A : \mu_{telephone} - \mu_{novel} >$, the p-value is 0.5%. There is **no** evidence to suggest that doing boring activities **increases** creativity.

(E) $H_0 : \mu_{telephone} - \mu_{novel} \geq 0.97$ vs $H_A : \mu_{telephone} - \mu_{novel} < 0.97$, the p-value is 0.5%. There is evidence to suggest that doing boring activities **increases** creativity.

(2) Social media companies believe that doing boring activities **decreases** creativity. What is the hypothesis of interest and using the above data the result of the test.

(A) $H_0 : \mu_{telephone} - \mu_{novel} \leq 0$ vs $H_A : \mu_{telephone} - \mu_{novel} > 0$, the p-value is 0.5%. The data suggests that doing boring activities **decreases** creativity.

(B) $H_0 : \mu_{telephone} - \mu_{novel} \leq 0$ vs $H_A : \mu_{telephone} - \mu_{novel} > 0$, the p-value is 99.5%. The data suggests doing boring activities **decreases** creativity.

(C) $H_0 : \mu_{telephone} - \mu_{novel} \geq 0$ vs $H_A : \mu_{telephone} - \mu_{novel} < 0$, the p-value is 99.5%. There **no** evidence to suggest that doing boring activities **decreases** creativity.

(D) $H_0 : \mu_{telephone} - \mu_{novel} \geq 0.97$ vs $H_A : \mu_{telephone} - \mu_{novel} < 0.97$, the p-value is 99.5%. The data suggests that doing boring activities **decreases** creativity.

(E) $H_0 : \mu_{telephone} - \mu_{novel} \geq 0$ vs $H_A : \mu_{telephone} - \mu_{novel} < 0$, the p-value is 0.5%. There is **no** evidence to suggest that doing boring activities **decreases** creativity.

(3) Psychologists want to test if there is a difference in creativity between those who read the telephone book and those who read a novel. What is the hypothesis of interest and based on the above the result of the test?

(A) $H_0 : \mu_{telephone} - \mu_{novel} \neq 0$ vs $H_A : \mu_{telephone} - \mu_{novel} = 0$, the p-value is 99%. The data suggests that doing boring activities **decreases** creativity.

(B) $H_0 : \mu_{telephone} - \mu_{novel} = 0$ vs $H_A : \mu_{telephone} - \mu_{novel} \neq 0$, the p-value is 99%. The data suggests there is a difference.

(C) $H_0 : \mu_{telephone} - \mu_{novel} = 0$ vs $H_A : \mu_{telephone} - \mu_{novel} \neq 0$, the p-value is 99%. The data **does not** suggest that there is a difference.

(D) $H_0 : \mu_{telephone} - \mu_{novel} = 0$ vs $H_A : \mu_{telephone} - \mu_{novel} \neq 0$, the p-value is 1%. The data suggests that there is a difference.

(4) Based on the data, construct a **99**% confidence interval for the mean difference. A t-table is given below for 57.48 df.

probability	0.3	0.15	0.10	0.05	0.025	0.01	0.005
t^*	0.527	1.04	1.30	1.67	2.00	2.39	2.67

(A) $[0.97 \pm 2.00 \times 0.36]$ (B) $[0.97 \pm 2.39 \times 0.36]$ (C) $[0.97 \pm 2.67 \times 0.36]$

(D) $[0.97 \pm 2.67 \times 0.36/\sqrt{60}]$ (E) $[0.97 \pm 2.39 \times 57.48/\sqrt{60}]$

(5) Fill in the blanks:

We use the t-distribution instead of the normal distribution when we ____(1)____.

For very large sample sizes, the confidence intervals constructed using the normal distribution and the t-distribution are ____(2)____

The t-distribution ____(3)____ reduce the skewness of data.

	(1)	(2)	(3)
A	if the data is skewed	almost the same	can
B	if the data has heavy tails	narrower	cannot
C	estimate the standard deviation	almost the same	cannot
D	if the data is skewed	wider	can
E	estimate the standard deviation	is very different	cannot

(6) In a randomized trial the efficacy (think of this as the mean) of a vaccine based on 20,000 randomly selected participants was found to be $90\% \pm 4\%$ with 99% confidence. Based on this data, a news outlet makes the following claim "Vaccine trial suggests that the vaccine efficacy is over 85%". Which statement is correct?

(A) The news outlet is **incorrect**. As the sample size is very small as compared to the 8 billion people in the world, the claim appears to be rather dubious. We have no idea of what the efficacy of the vaccine is, it could well be below 85%.

(B) The news outlet is **correct**. As 85% and over lies in the 99% confidence interval $[86, 94]\%$, the data suggests the true efficacy is over 85%.

(7) In a randomized trial the efficacy (think of this as the mean) of a vaccine based on 20,000 randomly selected participants was found to be $90\% \pm 4\%$ with 99% confidence. Based on this data, Breitbart news makes the following claim "Vaccine trial inconclusive, there is too much error". Which statement is correct?

(A) The news outlet is **incorrect**. The confidence interval takes into accounts the sample size in the construction of the margin of error. The results of the trial do suggest that the efficacy lies somewhere in the interval $[86, 94]\%$.

(B) The news outlet is **correct**. As the sample size is very small as compared to the 8 billion people in the world, the results of the trial appear to be rather dubious. We have no idea of what the efficacy of the vaccine is, it could well be below 85%.

(8-10) Oxometers are instruments for measuring the oxygen saturation level of blood in patients (the maximum saturation level is 100%). It is used in emergency rooms to determine whether a (Covid) patient should be admitted. A saturation level of over 90% is considered healthy, whereas saturation of 90% and below is considered low (and the patient is admitted). Oxometers are only an estimate of the oxygen saturation level (it is based on sending beams of red and infrared light through tissue at the tip of the finger). An accurate measurement of saturation is based on a blood sample.

Scientists suspect that (sick) dark skinned patients are getting oxometer readings of **over 90%** (due to different absorption of red and infra red light), even though their blood samples show they they have low oxygen saturation levels. To test this, a random sample of 40 dark skinned patients (whose blood readings show low oxygen saturation) have the oxometer reading taken. The average (sample mean) oxometer reading of these 40 patients is 91.12%

(8) What is the Scientists' hypothesis of interest?

(A) $H_0 : \mu \geq 90\%$ vs $H_A : \mu < 90\%$ (B) $H_0 : \mu = 90\%$ vs $H_A : \mu \neq 90\%$

(C) $H_0 : \mu \geq 91.12\%$ vs $H_A : \mu < 91.12\%$ (D) $H_0 : \mu \leq 91.12\%$ vs $H_A : \mu > 91.12\%$

(E) $H_0 : \mu \leq 90\%$ vs $H_A : \mu > 90\%$

(9) The data in the oxometer study described in Q9 is given in test below

One sample T hypothesis test:
μ : Mean of variable
$H_0 : \mu = 90$
$H_A : \mu > 90$

Hypothesis test results:

Variable	Sample Mean	Std. Err.	DF	T-Stat	P-value
Oxometer	91.122329	0.38930638	40	2.882895	0.0032

hypothesis $H_0 : \mu \leq 90\%$ vs $H_A : \mu > 90\%$ at the 5% level. Which statement is correct (this question continues to the next page)?

(A) The p-value is between 99-99.5%, there is evidence to suggest that the mean reading of the oxometer (for unhealthy, dark skinned patients) is **under** 90%.

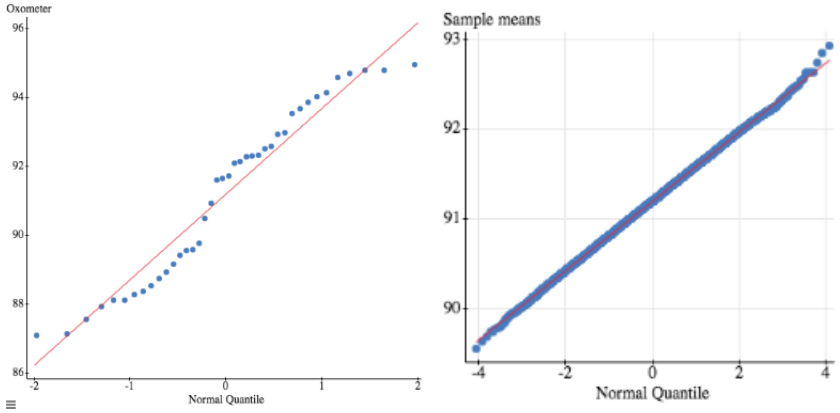
(B) The p-value is between 0.25-0.5%, there is evidence to suggest that the mean reading of the oxometer (for unhealthy, dark skinned patients) is **over** 90%.

(C) The p-value is 0.00%, there is evidence to suggest that the mean reading of the oxometer (for unhealthy, dark skinned patients) is **under** 90%.

(D) The p-value is 10.5%, there is **no** evidence to suggest that the mean reading of the oxometer (for unhealthy, dark skinned patients) is **over** 90%.

(E) The p-value is 90%, there is **no** evidence to suggest that the mean reading of the oxometer (for unhealthy, dark skinned patients) is **over** 90%.

(10) The QQplot of the oxometer data (described in Q8 and Q9) (see the left hand plot) and the sample mean (based on a sample size 40) (see right hand plot) is given below.



Based on these plots which statement is correct?

(A) Neither the actual data nor the distribution of the sample mean appears to be close to normal. The p-value in (Q9) is not reliable.

(B) The QQplot clearly shows the data is skewed. The p-value given in (Q9) was calculated incorrectly, it was far too large.

(C) The QQplot clearly shows the data is skewed. The p-value given in (Q9) was calculated incorrectly, it was far too small.

(D) The actual data does not appear to be too far from normality (though it is hard to quantify). The distribution of the sample mean looks very close to normal. The p-value given in (Q9) is reliable.

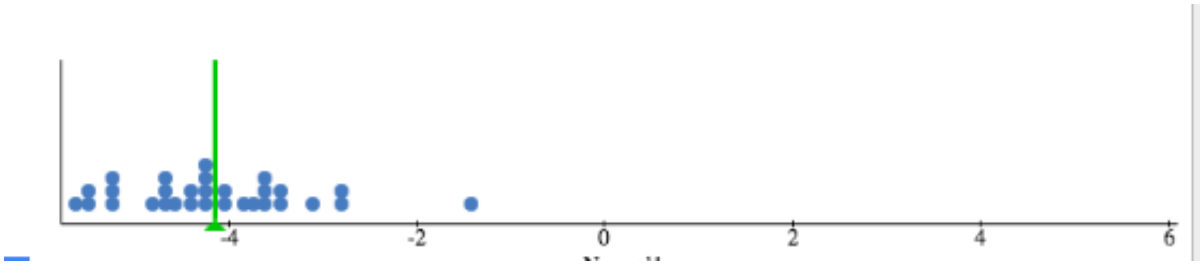
(11) The **maximum** weight a delivery truck can carry is **1000kg**.

The weight of an electrical appliance is normally distributed with mean $\mu = 52.5$kg and standard deviation $\sigma = 20$kg.

16 appliances are loaded into the truck, what is the probability that the truck will **not exceed** the maximum weight?

(A) 100% (B) 90.4% (C) 30.8% (D) 69.14% (E) 97.7%

(12) A data set is plotted below (the vertical line is the sample mean).



Let μ denote the population mean. We test the hypothesis $H_0 : \mu \geq 4$ vs $H_A : \mu < 4$ (the test is done at the 5% level). Based on the data which statement is correct?

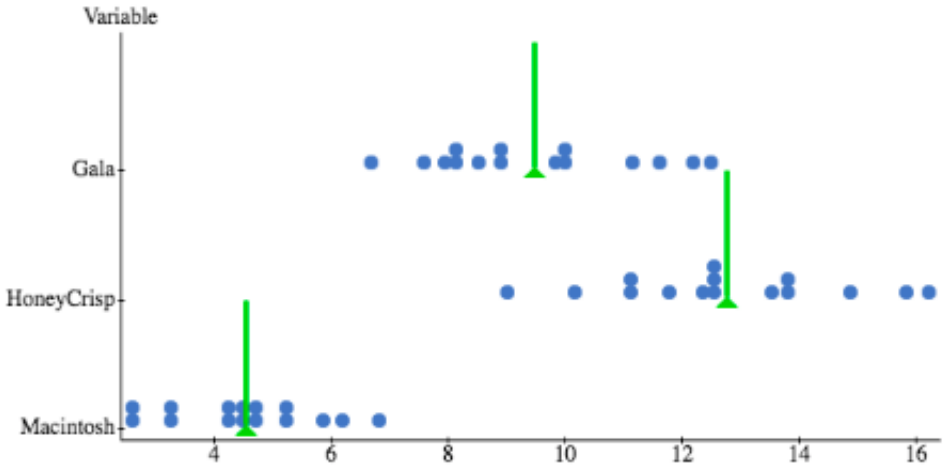
(A) The p-value is **less** than 0.1%. We reject the null, there **is** evidence the mean is **less than 4**.

(B) The p-value is **greater** than 5%. We reject the null, there is evidence the mean **is less than 4**.

(C) The p-value is **greater** than 5%, there is **no** evidence the mean **is less than 4**.

(D) The p-value is close to 100%, the null is true.

(13) The American Apple Association wants to compare different apple varieties. They randomly sample 15 apples each from Gala, Honey Crisp and Mcintosh varieties. A dot plot of the apples is given below (the vertical lines are the sample means for each group).



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Top: Gala, Middle: HoneyCrips and Bottom: Macintosh.

Use the figure above to identify the correct p-value for each hypothesis. Read each hypothesis **carefully**.

	$H_0 : \mu_{\text{Gala}} - \mu_{\text{Macintosh}} \leq 0$ $H_A : \mu_{\text{Gala}} - \mu_{\text{Macintosh}} > 0$	$H_0 : \mu_{\text{HoneyCrips}} - \mu_{\text{Mcintosh}} \leq 0$ $H_A : \mu_{\text{HoneyCrips}} - \mu_{\text{Macintosh}} > 0$	$H_0 : \mu_{\text{HoneyCrisp}} - \mu_{\text{Gala}} \leq 0$ $H_A : \mu_{\text{HoneyCrisp}} - \mu_{\text{Gala}} > 0$
A	20-30%	20-30%	20-30%
B	1-10%	less than 0.5%	less than 0.5%
C	greater than 99.5%	greater than 99.5%	between 90-99%
D	less than 0.5%	less than 0.5%	1-10%
E	10-20%	20-30%	30-40%

(14) Muriel Bristol claims she can detect which was poured first in her cup of tea, either milk or black tea. Fisher checks this claim by making 10 cups of tea, 5 where the milk was poured first and 5 where the milk was poured first. Muriel was asked to identify whether tea or milk was poured first in each cup of tea (and told 5 cups are with tea is poured first and 5 with the milk poured first).

We test the hypothesis H_0 : Muriel was guessing vs H_A : Muriel knew her tea. She correctly identified all 10 cups of tea. The chance of her identifying all the cups correctly by simply guessing is $1/252 = 0.4\%$. We do the test at the 5% level. Which statement is correct?

(A) The probability of the null being true is 0.4% and the probability the alternative is true is 99.6%. This suggest the alternative is true and she knew her tea.

(B) The p-value is 0.4%. This is below 5%, there is evidence to suggest she knew her tea.

(C) The p-value is 99.6%. This is above 5%, she was probably guessing.

(D) The p-value is 0.4%. This is below 5%, the data suggests she was guessing.

(15) Consider the hypothesis H_0 : innocent vs H_A : guilty, used in a criminal court. Evidence is collected and the p-value based on the evidence (under the null) is calculated. We reject the null if the p-value is less than a certain probability level, say 5%. If we reduce 5% to 1% what happens the proportion of innocent and guilty people who are convicted?

(A) Only 1% of innocent people **will be** to convicted and 5% of guilty people people will be convicted.

(B) Only 1% of guilty people **will be** convicted, but fewer innocent people will be convicted.

(C) Only 1% of innocent people **will be** convicted, but fewer guilty people are convicted.

(D) 1% of innocent **and** guilty people **will be** to convicted.

(E) 99% of guilty people **will be** to convicted and 1% of innocent people will be convicted.