

Solutions

Midterm 3 - STAT 301  
Spring 2015

Name:

UIN:

Signature:

Version B:

1. Do not open this test until told to do so.
2. This is a closed book examination, However you may use the cheat sheet provided and the tables you have brought with you. You should have no other printed or written material with you on the exam. But scrap paper is allowed.
3. You have 60 minutes to work on this exam. There are 15 multiple choice questions.
4. On the scantron please state the version of exam that you have.
5. You may use a calculator in the exam.
6. If there is no correct answer or if multiple answers are correct, select the **best** answer.
7. If you are unsure of what a question is asking for, do not hesitate to ask the instructor or course assistant for clarification.
8. Do all tests at the 5% unless specified otherwise.
9. Please only give one answer per question (the one that is closest to the solution).
10. Good Luck!!!

(1) It is important to know ones horse, in order to ensure that she is healthy. The pulse of a horse will vary from reading to reading but over time we can learn the mean resting pulse and standard deviation of any particular horse. It can be assumed that the pulse of a horse follows a normal distribution with mean  $\mu$  and standard deviation  $\sigma$ . An 'elevated' (high) resting pulse rate can be indicative of an underlying health problem. Debbie knows that the mean resting pulse of her horse (when it is healthy) is  $\mu = 40$  beats per second and the standard deviation is  $\sigma = 5$ . Debbie takes three pulse readings, which are 42, 45, 45.5 the sample mean (average) is  $\bar{x} = 44.167$ . Is there any evidence that the pulse of Debbie's horse is higher than normal? State the hypothesis of interest and do the test at the 5% level (remember to use the normal distribution).

(A)  $H_0 : \mu \leq 40$  vs  $H_A : \mu > 40$ , the p-value is **92.5%**, there is no evidence at the 5% level to suggest that the horse has an elevated pulse.

(B)  $H_0 : \mu \geq 40$  vs  $H_A : \mu < 40$ , the p-value is **92.5%**, there is no evidence at the 5% level to suggest that the horse has an elevated pulse.

(C)  $H_0 : \mu \leq 40$  vs  $H_A : \mu > 40$ , the p-value is **20.2%**, there is no evidence at the 5% level to suggest that the horse has an elevated pulse.

① (D)  $H_0 : \mu \leq 40$  vs  $H_A : \mu > 40$ , the p-value is **7.49%**, there is no evidence at the 5% level to suggest that the horse has an elevated pulse.

(E)  $H_0 : \mu = 40$  vs  $H_A : \mu \neq 40$ , the p-value is **1.59%**, there is evidence at the 5% level to suggest that the horse has an elevated pulse.

(2) Let us return to Debbie's horse in (Q1) and testing for an elevated pulse (as described in (Q1)). Three pulse readings are taken and the average/sample mean, denoted as  $\bar{x}$ , corresponds to the p-value 0.1%. Which statement(s) is/are correct?

$\frac{1}{3}$  (A) The sample mean must be greater than 48.8 (ie.  $\bar{x} > 48.8$ ).

$\frac{1}{3}$  (B) There is evidence at the 5% level to suggest that the Debbie's horse has an elevated pulse.

$\frac{1}{3}$  (C) The chance of getting the observed sample mean,  $\bar{x}$ , when Debbie's horse is healthy is extremely small.

① (D) [A], [B] and [C].

$\frac{2}{3}$  (E) [B] and [C].

(3) It is known that the mean resting pulse and standard deviation of Sidney the champion horse is  $\mu = 30$  and  $\sigma = 6$ . His owner checks for an elevated pulse (higher than the mean level) by taking **two** readings and does a statistical test at the 5% level.

Sidney's pulse is considered extremely high if his mean pulse level gets above  $\mu = 50$ . Using the plot in Figure 1, what is the chance of an extremely high pulse NOT being detected?

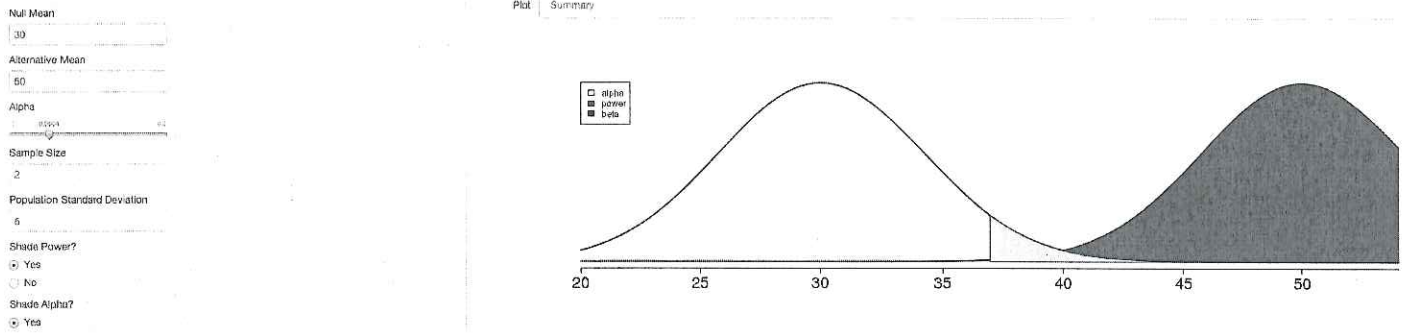


Figure 1: Power demonstration

- (A) <sup>1/2</sup> More than 95% (B) 95% (C) 50% (D) 5% (E) <sup>①</sup> Less than 5%.

(4) Using the plots in Figure 2, determine the **sample size required** such that the chance the test described in (Q3) ( $H_0 : \mu \leq 30$  vs  $H_A : \mu > 30$ ) can detect an elevated mean level of  $\mu = 42$  or more, is at least 99%?

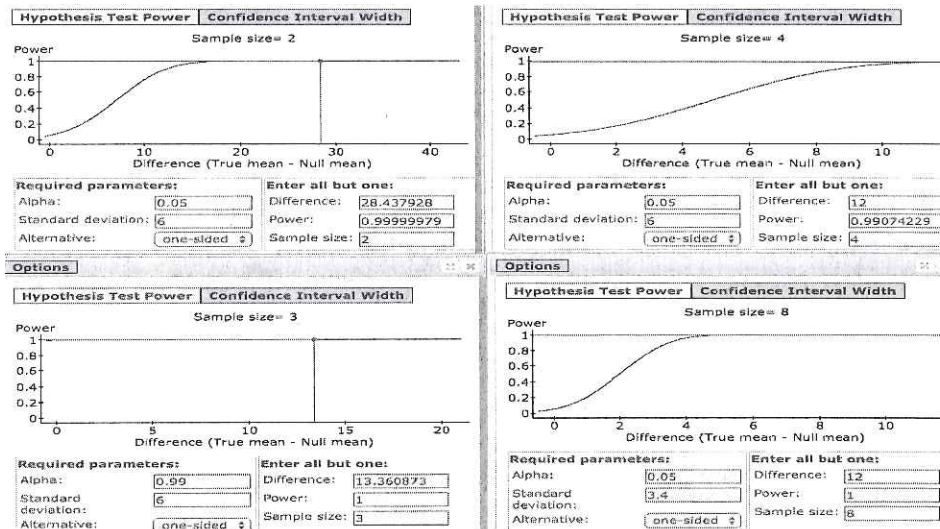


Figure 2: Power calculation

- (A) 8 (B) <sup>①</sup> 4 (C) 3 (D) 2 (E) 1.

(Q5) Match the following three experiment with the correct test.

1. To test the efficacy of a memory enhancing drug, 20 randomly sampled twins were analysed. One twin was placed in the Placebo group while the other was placed in the drug group (altogether there are 10 in the placebo group and 10 in the drug group).
2. To see whether the protein content in bread decreases with age, 30 loaves of breaded were baked. Immediately after baking the amount of protein was measure in each bread and then three days later the amount of protein was measured.
3. To see whether the protein content in bread decreases with age, 60 loaves of breaded were baked. They split the loaves into two groups, each of size 30. In the first group the amount of protein was measured immediately after baking. In the second group the amount of protein was measured 3 days after baking.

	[1]	[2]	[3]
(A)	Matched paired t-test	Independent Sample t-test	ANOVA
(B)	Test on proportions	Matched paired t-test	Matched paired t-test
(C)	Test on proportions	Independent Sample t-test	ANOVA
(D)	Matched paired t-test	Matched paired t-test	Independent sample t-test
(E)	Test on proportions	Independent sample	ANOVA

(6) It is believed that ownership of horses is different between Texas and California. Let  $p_T$  denote the **proportion** of Texans who own a horse,  $p_C$  the **proportion** of Californians who own a horse,  $\mu_T$  the **number** of Texans who own a horse and  $\mu_C$  the **number** of Californians who own a horse. Which hypothesis should we choose?

- (A)  $H_0 : p_T - p_C = 0$  against  $H_A : p_T - p_C \neq 0$ .
- (B)  $H_0 : \mu_T - \mu_C \neq 0$  against  $H_A : \mu_T - \mu_C = 0$ .
- (C)  $H_0 : \mu_T - \mu_C = 0$  against  $H_A : \mu_T - \mu_C \neq 0$ .
- (D)  $H_0 : p_T - p_C \leq 0$  against  $H_A : p_T - p_C > 0$ .
- (E)  $H_0 : \mu_T - \mu_C \leq 0$  against  $H_A : \mu_T - \mu_C > 0$ .

(7) Behcet's syndrome is an autoimmune disorder, where oral ulcers is one symptom. Recently a new drug, called Apremilast, for controlling the oral ulcers has been developed. A phase two clinical trial was conducted, where 111 patients with Behcet's syndrome, were randomly assigned to receive Apremilast or a placebo, twice daily for 12 weeks. The trial was fully randomised, in the sense that neither patients or clinicians were aware of the study assignments. The number of oral ulcers in the mouth of each patient was measured at the end of the 12 weeks trial. The following findings were reported in the New England Journal of Medicine:

'The mean number of oral ulcers per patient at week 12 was lower in the Apremilast group than the Placebo group (the 95% confidence intervals for the respective means giving  $0.5 \pm 1.0$  vs  $2.1 \pm 2.6$ ), the result of an independent sample t-test gave  $P\text{-value} < 0.1\%$ .' Let  $\mu_A$  and  $\mu_P$  denote the mean number of ulcers for Apremilast and the placebo, respectively,

What can we conclude from the study?

- (A) Using a fully randomised trial ensures normality of the data.
- (B) In the test  $H_0 : \mu_A - \mu_P \geq 0$  vs  $H_A : \mu_A - \mu_P < 0$ , the  $p\text{-value} < 0.1\%$ , thus there is no evidence to suggest that Apremilast may reduce oral ulcers.
- (C) In the test  $H_0 : \mu_A - \mu_P \geq 0$  vs  $H_A : \mu_A - \mu_P < 0$ , the  $p\text{-value} < 0.1\%$ , thus there is strong evidence to suggest that Apremilast may reduce oral ulcers.
- (D) In the test  $H_0 : \mu_P - \mu_A \geq 0$  vs  $H_A : \mu_P - \mu_A < 0$ , the  $p\text{-value} < 0.1\%$ , thus there is evidence to suggest that Apremilast may increase oral ulcers.
- (E) [A] and [D].

(8-9) It is believed that gender may have some influence (though it is not specified which way) on the length of a depressive episode. The length of depression for 109 depressive patients was analyzed (38 were female and the rest were male). The data is summarized below (you will need this to answer Q8 and Q9).

$\mu_1$  : Mean of Male  
 $\mu_2$  : Mean of Female  
 $\mu_1 - \mu_2$  : Difference between two means  
 (without pooled variances)

	Difference	Sample Diff.	Std. Err.	DF	probability	0.3	0.15	0.10	0.05	0.025	0.01	0.005
$\mu_1 - \mu_2$	-2.93106	16.811396	89.97624		$t^*$	0.52	1.04	1.29	1.695	1.98	2.37	2.63

Critical values for a t-distribution with 89.97 degrees of freedom.

(8) What is the test of interest and the results of the test?

① (A)  $H_0 : \mu_M - \mu_F = 0$  vs  $H_A : \mu_M - \mu_F \neq 0$ . The p-value is over 60%, there is **no** evidence in the data to suggest that gender has an influence on the length of a depressive episode.

(B)  $H_0 : \mu_M - \mu_F \geq 0$  vs  $H_A : \mu_M - \mu_F < 0$ . The p-value is over 60%, there is evidence in the data to suggest that males suffer longer depressive episodes than females.

(C)  $H_0 : \mu_M - \mu_F = 0$  vs  $H_A : \mu_M - \mu_F \neq 0$ . The p-value is over 60%, there is evidence in the data to suggest that gender has an influence on the length of a depressive episode.

(D)  $H_0 : \mu_M - \mu_F \geq 0$  vs  $H_A : \mu_M - \mu_F < 0$ . The p-value is less than 4%, there is some evidence in the data to suggest that females suffer longer depressive episodes than males.

(E)  $H_0 : \mu_M - \mu_F \leq 0$  vs  $H_A : \mu_M - \mu_F > 0$ . The p-value is over 60%, there is evidence in the data to suggest that females suffer longer depressive episodes than males.

(9) Construct a **99%** confidence interval for the mean difference in lengths of male and female depressive episodes.

(A)  $[-2.9 \pm 1.04 \times 16.81]$  (B)  $[-2.9 \pm 2.63 \times \frac{16.81}{\sqrt{109}}]$  (C)  $[-2.9 \pm 2.37 \times 16.81]$

(D)  $[-2.9 \pm 0.95 \times 16.81]$  (E)  $[-2.9 \pm 2.63 \times 16.81]$ .

(10) 30 people were put on a weight loss program. It was found that after 20 weeks the average (sample mean) weight loss was 13 pounds. A 95% confidence interval for the mean weight loss is  $[13 \pm 5] = [8, 18]$ .

Blue Cross Blue Shields, the health insurance company, will only pay for the weight loss program if there is **evidence to suggest** the mean weight loss is over 16 pounds. Based on the data will the insurance fund the scheme?

(A) Since 16-18 is in the 95% CI there is evidence to suggest that the mean weight loss is over 16 pounds.

① (B) There is no evidence to suggest that mean weight loss is over 16 pounds.

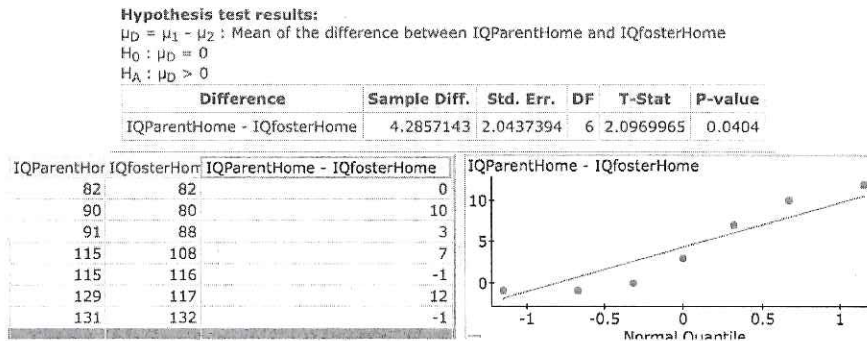
(C) It is impossible to be sure without doing a formal statistical test and calculating the p-value.

(D) [B] and [C] (E) [A] and [C].

②  $\frac{1}{2}$

- (11) Does environment have an influence on 'intelligence'? It has been conjectured that being brought up in a Foster home may reduce the intelligence as compared to being brought up in parental home.

To test this hypothesis, 7 twins who were separated at birth, one being brought up in the parental home and the other in a foster home, had their IQ assessed. The results of the test, the data and QQplot of the data are given below. Let  $\mu_F$  and  $\mu_P$  denotes the population mean when being brought up in the foster and parental homes, respectively.



- (A)  $H_0 : \mu_P - \mu_F \leq 0$  vs  $H_A : \mu_P - \mu_F > 0$ , the p-value is 4.04% hence there is some evidence at the 5% level to conclude that being cared for in the parental home has a positive effect on IQ.

- (B)  $H_0 : \mu_P - \mu_F \leq 0$  vs  $H_A : \mu_P - \mu_F > 0$ , the p-value is 4.04%, there is no evidence to suggest that parental home has an influence on IQ.

- (C) The data is not normal (it is integer valued) and the sample size is small, therefore the p-value will not be very reliable and we need to careful interpreting the p-value.

- (D) [B] and [C]      (E) [A] and [C]

- (12) 55% of the population are known to recycle on a regular basis. It is believed that 'young people' (defined as people below the age of 25) **tend to recycle more than the general population**. A random sample of 300 young people were questioned, and asked whether they recycle on a regular basis. 180 of the 300 said that they did (60% of the sample). Let  $p$  denote the proportion of young people who regularly recycle. What is the hypothesis of interest and do the test at the 5% level. You will need to use the output given in Figure 3

- (A)  $H_0 : p \leq 0.55$  vs  $H_A : p > 0.55$ , the p-value is more than 95%, there is no evidence (in the data) to suggest that young people recycle more than the general population.

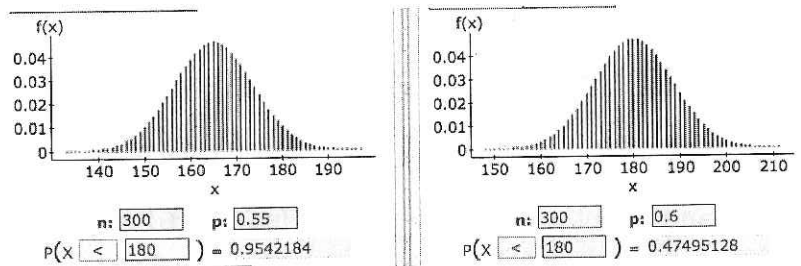


Figure 3: Output for Q12

- (B)  $H_0 : p \leq 0.5$  vs.  $H_A : p > 0.5$ , the p-value is less than 1%, so there is some evidence to suggest that the majority of the population support recycling.
- (C)  $H_0 : p \leq 0.6$  vs  $H_A : p > 0.6$ , the p-value is about 53%, there is no evidence to suggest that young people recycle more than the general population.
- (D)  $H_0 : p \leq 0.6$  vs  $H_A : p > 0.6$ , the p-value is about 47.5%, there is no evidence to suggest that young people recycle more than the general population.
- (E)  $H_0 : p \leq 0.55$  vs  $H_A : p > 0.55$ , the p-value is 4.6%, there is some evidence (in the data) to suggest that young people recycle more than the general population.
- (13) Using the output in Figure 4 construct a 95% CI for the mean proportion of young people who support recycling.

**95% confidence interval results:**  
 p : Proportion of successes  
 Method: Standard-Wald

Proportion	Count	Total	Sample Prop.	Std. Err.
p	180	300	0.6	0.028284271

Figure 4: Partial output for 95% CI

- (A) Based on the data with 95% confidence the proportion of young people who recycle is between [54.5, 65.5]%
- (B) Based on the data with 95% confidence the proportion of young people who recycle is between [44.8, 55.4]%
- (C) Based on the data with 95% confidence the proportion of young people who recycle is between [44.8, 55.4]%
- (D) Based on the data with 95% confidence the proportion of young people who recycle is between [59.6, 60.3]%



- (14) The proportion of farm animals with foot and mouth disease was 1% (0.01). New measures have been introduced to reduce infection. The Department of Agriculture wants to know whether they were effective in reducing incidence (dropping the proportion below 1%). A random sample of 200 sheep was taken and it was found that one of them had foot and mouth disease. The test was done using both the binomial distribution and also using the normal distribution. The output for the test  $H_0 : p \geq 0.01$  against  $H_A : p < 0.01$  is given in Figure 5. Which comment(s) is/are correct?

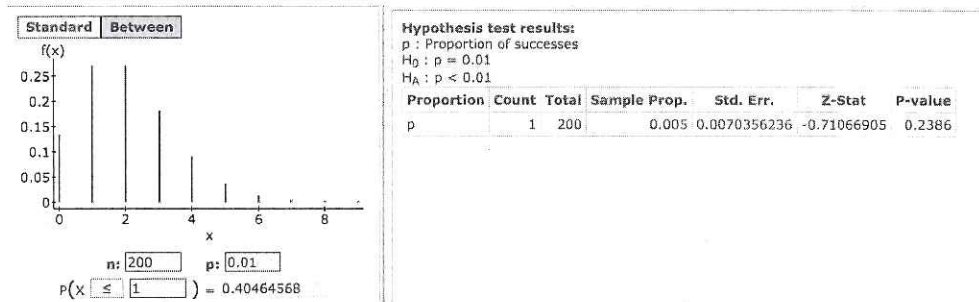


Figure 5: Foot and mouth disease test.

- (A) The different methods give conflicting results. The binomial distribution suggests a reduction in the proportion of cases, whereas the normal distribution does not suggest a reduction in the proportion of cases.
- $\frac{1}{2}$  (B) The discrepancy between p-values is because  $p$  is so small that the binomial distribution is highly skewed. Thus the normal approximation is not reliable.
- $\frac{1}{2}$  (C) Regardless of what method is used, there is no evidence of a reduction in the proportion of foot and mouth cases.
- (D) [A] and [B]
- $\frac{1}{1}$  (E) [B] and [C].
- (15) It is known that the number of people who develop an adverse reaction to a pain killer is at most 20% of the population (this is an upper bound). What is the minimum sample size required to ensure that the Margin of Error for a 95% CI is at **most** 1%?
- (A) 1230 (B) 3460 (C) 6147 (D) 9604 10145.

