13 Tests for White Noise

The results above for $\hat{\rho}$ and $\hat{f}$ can be used to form tests for white noise.

Thus suppose that $X(1), \ldots, X(n)$ is a realization of length n from a $WN(\sigma^2)$ process. Then

1. The **Bartlett test** for white noise (see the BARTTEST command) is based on the cumulative periodogram and uses the fact that

$$\lim_{n\to\infty} \mathsf{P}(B > b) = 1 - \sum_{j=-\infty}^{\infty} (-1)^j e^{-2b^2 j^2},$$

   where $q = [n/2] + 1$ and the Bartlett test statistic B is given by

$$B = \max_{1 \le k \le q} \sqrt{q} \left| \hat{F}(\omega_k) - \frac{k}{q} \right|$$

   Thus B measures the largest deviation of $\hat{F}$ from the white noise line $y = 2x$. If we observe a value b of the statistic B, other possible values of B are "more extreme" than b if they are bigger than b (a larger deviation of $\hat{F}$ from the line). Thus the probability above is the p-value and we reject white noise if p-value $< \alpha$.

2. The **Q test** (also called portmanteau test; see the QTEST command) for white noise is based on the fact that for WN the $\hat{\rho}$'s should be small and the fact that

$$\lim_{n\to\infty} \mathsf{P}(Q > q) = \mathsf{P}(\chi^2_m > q),$$

where the Q statistic is given by

$$Q = n(n+2)\sum_{j=1}^{m} \frac{1}{n-j}\hat{\rho}^2(j).$$

Thus Q measures how big a weighted sum of squares of the first m (which must be chosen) $\hat{\rho}$'s is. The bigger Q is the more evidence we have against the white noise null hypothesis. Thus the p-value is the probability given above, and we reject white noise if p-value $< \alpha$. The question arises as to what value of m should we use in this test. The usual answer to this question is to try several values (such as 10, 20, 40, for example) and see if the conclusion reached changes for different values. If not, then the conclusion to reach is clear, whereas if rejecting white noise or not changes for different m's, then the conclusion is that the data are close to white noise but perhaps not actually white noise.