Statistics 614 – Probability for Statistics
Section 600, Fall Term, 2014

This is a course in probability at the measure theoretic level, with emphasis both on understanding measure theory and its relationship to probability and on understanding the crucial roles that probability plays for statistics. Topics include probability measures, Lebesgue-Stieltjes integration, sigma-fields, random variables, expectation, moment inequalities, independence, convergence of random variables and sample moments, characteristic functions, convergence of distributions, the central limit theorem and the delta method, and conditional expectation. The intention is to lay a foundation of theory that will enhance the student's ability to read advanced probability and statistics literature and to write a dissertation.

Course Information

Time and Place: MWF 12:40pm–1:30pm, Blocker 448.

Instructor: Daren Cline. (http://stat.tamu.edu/~dcline)

Office: Blocker 459D, 845-1443.

E-mail: dcline@stat.tamu.edu

Office Hours: MWF 8:30am–10:00am or by appointment.

Course Web Page: http://stat.tamu.edu/~dcline/614.html. Lecture notes and homework assignments will be available at this site. Access to them will require a password that I will provide to you.

Text: S.I. Resnick, *A Probability Path*, Birkhäuser (*required*).

References: (on reserve in Evans Library)

- R.B. Ash, *Probability and Measure Theory*, Academic Press.
- K.B. Athreya and S.N. Lahiri, *Measure Theory and Probability Theory*, Springer.
- P. Billingsley, *Probability and Measure, 3rd ed.*, Wiley.
- R. Durrett, *Probability: Theory and Examples, 4th ed.*, Cambridge Univ. Press.
- E. Lukacs, *Characteristic Functions*, Griffin.
- A.N. Shiryayev, *Probability*, Springer.

Prerequisite: *Statistics 610 and Mathematics 409 (or 615) or their equivalent.*
The statistics requirement includes

- theory of probability distributions for random variables and random vectors,
- expectations, moments and variance,
- conditional distributions and conditional expectations,
- probability generating functions and moment generating functions,
- probability and moment inequalities,
- the law of large numbers and the central limit theorem.

The mathematics requirement is advanced calculus, specifically

- knowing how to produce careful, rigorous proofs,
- sequences, limits and series,
- continuity, differentiability and Taylor's expansion,
- uniform convergence and uniform continuity,
- integrals and power series, Fourier and Laplace transforms.

A previous course in measure theory is not required.

Homework: Homework will be assigned (on the course web page) and collected regularly. Homework is worth 30% of the total term score.
Please see the homework policy below.

Exams: One midterm exam worth 30% and a final exam worth 40%.
Please see the exam policy below.

Exam Dates: Midterm Exam: TBA.
Final Exam: TBA.

Course Information (cont.)

Disabilities Help:	The Americans with Disabilities Act ensures that students with disabilities have reasonable accommodation in their learning environment. If you have a disability and need help, please contact me and Disability Services in B118 Cain Hall, 845-1637.
Academic Integrity:	You are expected to maintain the highest integrity in your work for this class, consistent with the university rules on academic integrity. This includes not passing off anyone else's work as your own, even with their permission. Please see the homework and exam policies below for specifics.
Copyright:	Each document provided on my web pages or by me is copyrighted with all rights reserved, whether or not the document explicitly states so. They may only be used for academic purposes and they may not be reproduced or sold without my permission. You may refer to them for other classes or for research, just as you would any book, as long as you give proper credit and neither you nor anyone else reproduces them for sale or other distribution. To use some of the material for instruction purposes, you need to first get written permission from me (Daren Cline, TAMU Department of Statistics, College Station TX 77845-3143).

Course Policies

Homework Policy:

Your homework solutions must be your own work, not from outside sources, consistent with the university rules on academic integrity. I expect you to follow this policy scrupulously. *Your performance on the exams is much more likely to be better.* (Also, relying on others' solutions will cause me to think I can ask harder questions on the exams!)

You may use:

- Your textbook and notes from class.
- Your notes, homework, etc., from a related class that you took or are taking.
- References listed on the syllabus.
- Discussion with me.
- *Voluntary, mutual and cooperative* discussion with other students currently taking the class. This does not mean copying from each other.

You may not use:

- Solutions manuals (printed or electronic) other than what is provided with the required texts.
- Solutions from previous classes.
- Solutions, notes, homework, etc., from students who took the class previously.
- Solutions, notes, homework, etc., from classes taught elsewhere or at another time.
- Copying from students in this class, including expecting them to reveal their solutions in "discussion". That is, you may work together as indicated above as long as you prepare your own solutions.

Homework is to be submitted by the end of class on its due date unless I specify otherwise. *Late homework is not acceptable.*

Exam Policy:

Your exam solutions must be your own work, using only resources I explicitly allow, consistent with the university rules on academic integrity.

Each exam will be comprehensive and cumulative.

- Please bring your own paper. I ask that separate problems be on separate sheets.
- Bring resources (such as notes) only if I explicitly allow them.

I will not expect you to quote theorems and results explicitly but I do expect you to demonstrate that you can make correct use of them. Specifically, you will need to:

- Show all your work. This does not necessarily mean showing every individual algebraic or calculus step – but it must be clear what those steps would be.
- Identify (by number, name or description) any theorems, examples or homework problems you use.
- Verify conditions and assumptions as needed for those theorems and examples.
- Clearly identify the solution and/or the end of a proof or derivation.

No exam may taken early or made up, except if you provide a university excused absence with appropriate documentation.

Selected problems from my old exams will be available on the course web page. However, their content may not exactly match this semester's exams.

Makeup Policy:

This is based on university policy.

- If you must miss an exam due to illness or other university excused absence, notify me or the Statistics Department (before, if feasible, otherwise within two working days after you return). Contact me as soon as possible to schedule a make-up exam.
- An Incomplete will be given only in the event you have completed most of the course but circumstances beyond your control cause prolonged absence from class and the work cannot be made up.

Course Outline

Topic	Textbook Section
1. Events and Classes of Events	
outcomes, events, review of set theory	1.2
limits of events	1.3, 1.4
π-classes, fields and σ-fields	1.5, 1.6
Borel σ-fields	1.7, 1.8
Dynkin's π-λ class theorem	2.2
2. Probability Measures and Measures	
probability measures and general measures	2.1 – 2.3
properties of measures, probability distributions, uniqueness	2.1
distribution functions, Lebesgue-Stieltjes measures	
extension and existence theorems	2.4, 2.5
3. Random Variables and Measurability	
random variables and measurable functions, σ-field generated by a random variable	3.1 – 3.3
induced measures, distribution of a random variable	3.2
sufficient conditions for measurability	3.2, 5.1
4. Expectation and Integration	
definitions, consistency of definitions	5.1, 5.2, 5.4
properties of expectation and integrals	5.2
Lebesgue-Stieltjes integration, absolute continuity, Riemann integrals	5.6
monotone and dominated convergence theorems, extensions	5.2, 5.3
5. Advanced Integration	
equating differently defined integrals	5.5, 5.6
moments and inequalities, finiteness of moments	5.2
product spaces, multiple integration, Tonelli-Fubini theorem	5.7 – 5.9
characteristic functions, inversion formulas	9.1 – 9.5
6. Independence	
independence of finitely many events or random variables	4.1, 4.2
infinite collections of independent events or random variables	4.3, 4.4
Borel-Cantelli lemmas, tail events, Kolmogorov's 0-1 law	4.5
7. Convergence of Sequences of Random Variables	
modes of convergence and their relationships	6.1 – 6.3, 6.5
moment inequalities, Jensen's inequality	6.5
uniform integrability, convergence in mean	6.5, 6.6
8. The Law of Large Numbers and Convergence of Random Series	
strong law of large numbers	7.1, 7.4
Glivenko-Cantelli theorem	7.5
convergence of series	7.3, 7.6
9. Convergence of Distributions	
weak convergence, Scheffé's theorem	8.1, 8.2, 8.5
Slutsky's theorem, delta method, Skorohod's lemma, continuous mapping	8.3, 8.6
tightness, Prohorov's theorem, the continuity theorem	9.5, 9.6
portmanteau theorem, multivariate convergence	8.4
10. Weak Convergence for Sums and Maxima	
central limit theorem, Lindeberg-Feller and Lyapounov theorems	9.7, 9.8
convergence to types, infinitely divisible and stable distributions	8.7
extreme value distributions	8.7
11. Absolute Continuity and Conditional Expectation	
conditional expectation	10.2, 10.3
regular conditional distributions	
conditional independence and exchangeablilty	
absolute continuity, Radon-Nikodym theorem, Lebesgue decomposition	10.1