

The MH algorithm

The purpose of this note is to provide an illustration of the MH algorithm. Specifically, we aim to sample from a density

$$f(x) \propto \frac{1}{[1 + (x-1)^2/5]^3}, \quad x \in \mathbb{R}.$$

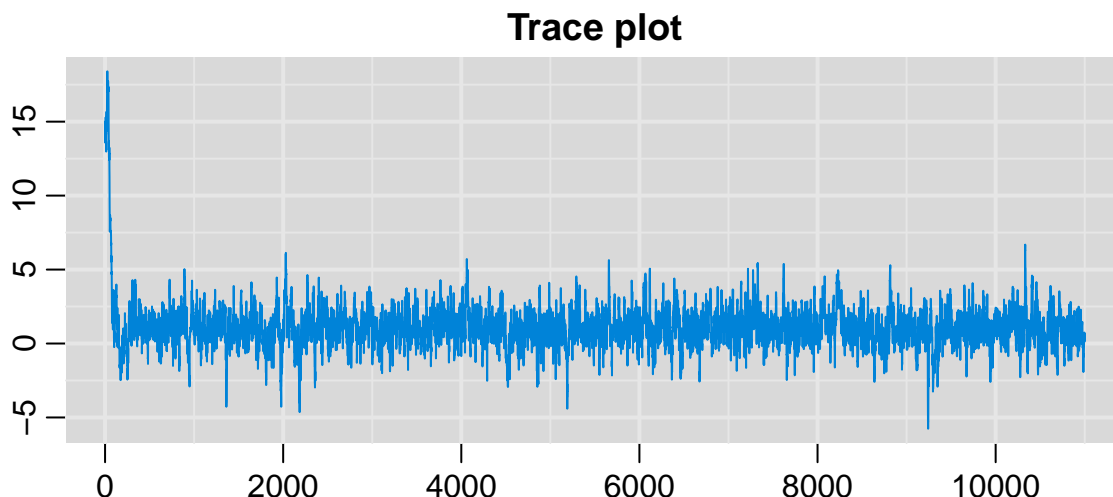
Clearly, this is a t_5 distribution centered at 1. We shall use a $N(0, \sigma_{MH}^2)$ proposal distribution. We run the chain for `MCMC = 10000` iterations. The following steps implements the MH algorithm.

```
set.seed(86)
m = 1           # center of t distribution
MCMC = 11000    # number of iterations
BURNIN = 1000   # number of initial samples to be discarded
sigMH = 1       # standard deviation of proposal
x = rep(0,MCMC) # save samples
accpind = rep(0,MCMC) # indicator of acceptance
xold = 15       # initialize

for (g in 1:MCMC){
  xnew = xold + sigMH*rnorm(1) #propose new value
  rat = ( (1+(xold-m)^2/5)/(1+(xnew-m)^2/5) )^3 # calculate MH ratio
  rat = min(1,rat)
  u = runif(1)
  x[g] = (u<rat)*xnew + (u>=rat)*xold
  accpind[g] = (abs(x[g]-xold) > 0) # 1 if proposal accepted
  xold = x[g]
}
```

First, let us plot the samples versus iteration number. Such plots are called trace plots.

```
library(coda)
library(mcmcplots)
xM = as.mcmc(x)
traplot(xM,main = "Trace plot")
```



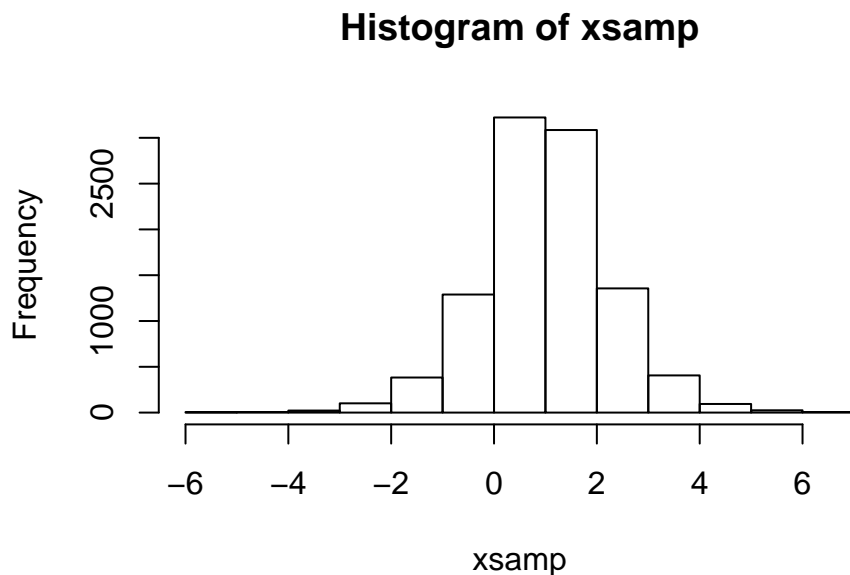
Let us now collect the samples post burn-in and calculate their mean and standard deviation. Recall that the mean and standard deviations of the target distribution are 1 and $\sqrt{5/3} = 1.29$ respectively.

```
xsamp = x[(BURNIN+1):MCMC]
c(mean(xsamp),sd(xsamp))
```

```
## [1] 1.004345 1.249585
```

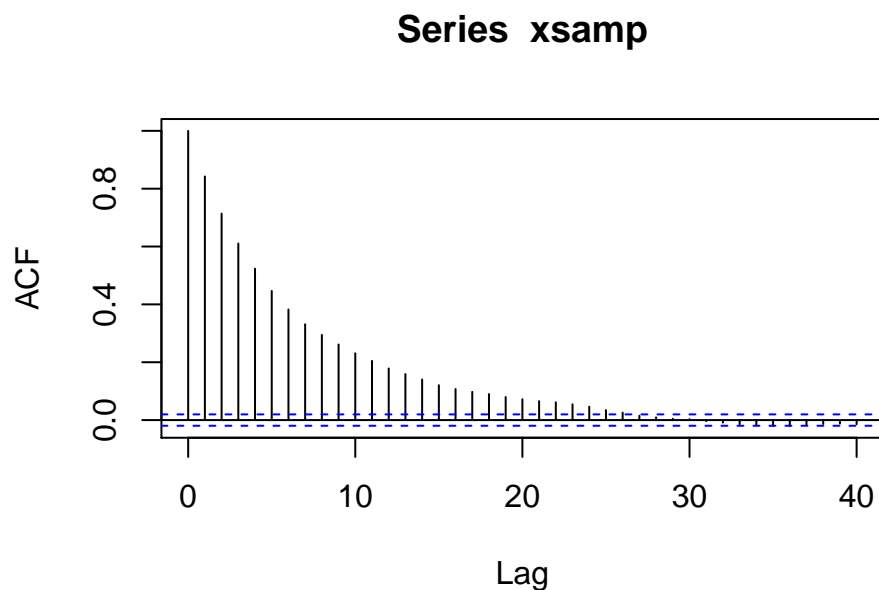
Let us also plot a histogram of the obtained samples.

```
hist(xsamp)
```



Next, we plot the auto-correlation function of the chain.

```
acf(xsamp)
```



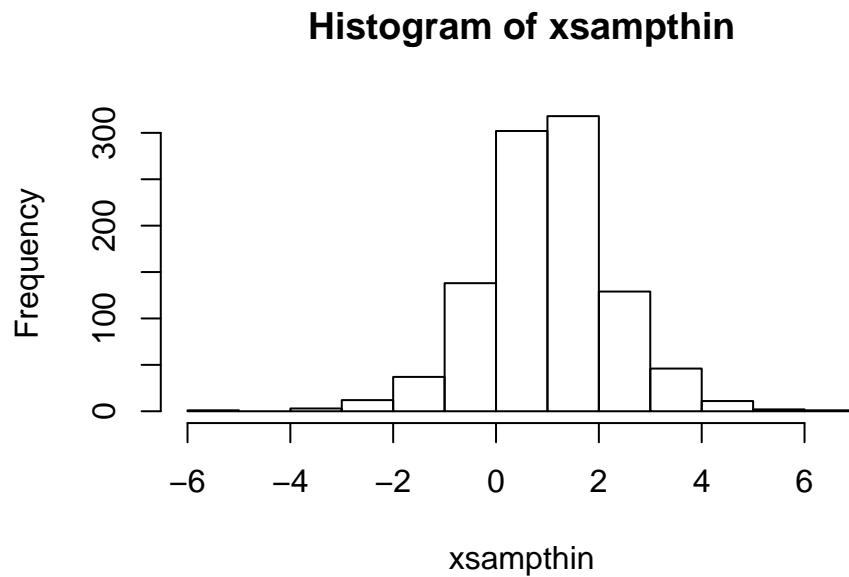
Let us now thin the chain. We collect every 10th sample post burn-in.

```
xsampthin = x[seq((BURNIN+1),MCMC,by=10)]  
c(mean(xsampthin),sd(xsampthin))
```

```
## [1] 1.014665 1.273118
```

PLot histogram of thinned samples and autocorrelation function.

```
hist(xsampthin)
```



```
acf(xsampthin)
```

